Resonant electromagnetic emission from intrinsic Josephson-junction stacks in a magnetic field

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We study the effect of an applied magnetic field on the resonant radiation from stacks of intrinsic Josephson junctions. We show that, when applying a moderate dc magnetic field in the plane of the junctions, a huge amplification of terahertz radiation can be obtained. This radiation enhancement occurs due to a modulation of the critical current. Furthermore, a dc magnetic field reducing the "radiation coupling" between junctions, produces a peculiar frequency-driven transition from stable to unstable terahertz radiation resonance.

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I. INTRODUCTION

Devices that could use the terahertz waveband would be important for potential applications.[1] However, the "terahertz gap" (from 0.5 to 10 THz) is hard to use because no practical sources of terahertz radiation exist.[2] Stacks of intrinsic Josephson junctions (SJJs), made of $Bi_2Sr_2CaCu_2O_8$ (BSCCO) or other layered high-T_c superconductors, are a candidate material that could fill the frequency gap from several tenths to ~ 2 THz.[3] A requirement for producing terahertz radiation from a SJJ is a synchronization of the high-frequency oscillations in all Josephson junctions (see, e.g., Ref. 4 and references therein). One approach to achieve such a synchronization is the application of a magnetic field to induce a coherent vortex flow.[3,5] However, the usual triangular vortex lattice produces a weak noncoherent radiation, while a rectangular lattice, which could in principle generate more terahertz power, is almost always unstable.[6,7]

An advance toward realizing superconducting terahertz emitters was recently made, when radiation power (up to 0.5 μW at frequencies $f=\omega/2\pi$ up to 0.85 THz) was measured coming out from BSCCO mesas.[8] In this experiment, a bias current I, with a density J higher than the critical value J_c, was initially applied to mesas along the c axis. Afterward, the current was decreased. When decreasing J below J_c, but still in the resistive state due to hysteresis, sharp peaks of terahertz emission were observed in the descending resistive branch of the current-voltage (IV) characteristic. The authors of Ref. 8 argued that the peaks in the irradiation power were observed under conditions of a cavity resonance.[9,10] That experiment[8] was performed in zero magnetic field and oscillations, with frequency $\omega_J=2e\Delta V/\hbar$, of the electromagnetic field in the sample appeared due to the standard ac Josephson effect; here ΔV is the voltage per single junction. Possibly, the synchronization of the field oscillations in different junctions might have been achieved due to "radiation coupling" (i.e., the radiation itself would induce the junctions to oscillate in phase) and would not require any external stabilization.[11,12] Note that a different way to produce radiation, using an artificial modulation of J_c, was proposed earlier in Refs. 13–16.

The idea to use cavity resonances in SJJs to attain a large output of the terahertz emission was intensively studied in many articles (see., e.g., Refs. 8, 12, and 17–19). In very earlier theoretical papers by Kulik,[20] it was shown that the application of a dc magnetic field in the plane of the contact produces to a drastic increase in the cavity resonance wave amplitude in the case of a single Josephson junction. This amplification, and the possible synchronized flux line motion, was studied by Kleiner *et al.*[21] in the case of SJJ. However, in Ref. 21, the analysis was done *only* numerically and the problem of the terahertz emission was *not* considered. The terahertz irradiation from a SJJ in a strong dc applied magnetic field, when cavity resonances occur, was also studied numerically in Ref. 19. The numerical analysis in Refs. 19 and 21 did not study the stability domain of the synchronized oscillations. Thus, the crucial problems of (i) the influence of weak magnetic fields on terahertz radiation and (ii) the stability of synchronized oscillations in SJJs has not been properly addressed so far. Both of these subjects are the focus of this paper.

Here we study *analytically* the effect of a weak in-plane dc magnetic field H_{dc}, on the radiation power output from BSCCO SJJ carrying an external bias current I along the c axis. The size of the SJJ is smaller than the c-axis London penetration depth λ_c and the triangular flux line lattice settles in the sample (suppressing the radiation) when $H_{dc} \gg H_{c1}$. In contrast, the application of a weak magnetic field H_{dc} gives rise to a modulation of the critical current density J_c in the SJJ and, similar to the predictions of Ref. 12, to a significant increase in the radiation power. A significant advantage of the approach to the problem using a dc magnetic field is that the J_c modulations are identical in all junctions (which is a major technical difficulty for producing SJJ with artificial modulations). In addition, the Josephson dynamics of SJJ in an external magnetic field provides interesting and rich possibilities to manage the radiation parameters by small changes in the magnetic field or bias current.[19] In particular, the coherent mode can be destroyed and restored by small variations in either the applied magnetic field or the bias current. An important part of the present study is the investigation of the domain of stability of the synchronized terahertz irradiation. We derive analytical conditions that allow us to obtain the limitations on the number N of contacts in the SJJ and on the value of the applied magnetic field H_{dc}. If

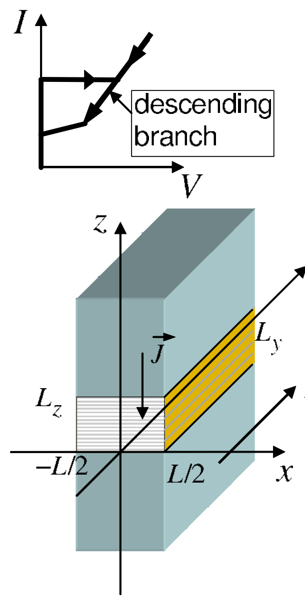

FIG. 1. (Color online) Sample geometry. The upper left curve schematically shows the current-voltage, *IV*, characteristics of the sample.

either N or H_{dc} exceed these limits, the synchronized oscillations would disappear.

II. RADIATION EMISSION

We consider a SJJ located between large superconducting leads with the same lateral dimensions as the stack, as in Fig. 1. We assume that the sizes of the sample in the x and z directions (L and L_z) are much smaller than along the y axis ($L, L_z \ll L_y$); also L, $L_z < \lambda_c$, and the number of junctions $N = L_z/s$ is large, $N \gg 1$; here $s(\sim 1.5$ nm) is the interlayer distance. We also assume that L, $L_z < c/\omega_J = 1/k_\omega < L_y$, which allows us to disregard the y- and z-coordinate dependence of the fields and only consider the radiation in the x direction. In the descending resistive branch of the *IV* curve (see left inset in Fig. 1) the current density $J < J_c = c\Phi_0/8\pi^2 s\lambda_c^2$, and we neglect the current dependence of the Josephson plasma frequency $\omega_p = c/\lambda_c\sqrt{\varepsilon}$, where ε is the dielectric constant. In this geometry, the electromagnetic field in the vacuum has the form $\mathbf{H} = (0, H_y, 0)$, $\mathbf{E} = (E_x, 0, E_z)$, and only outgoing waves propagate; that is

$$H, E_x, E_z \propto \exp(ik_x|x| + ik_z z - i\omega_J t).$$

Here $k_x = \sqrt{k_\omega^2 - k_z^2}$ for $k_\omega^2 > k_z^2$, and $k_x = i\sqrt{k_z^2 - k_\omega^2}$ for $k_\omega^2 < k_z^2$. This problem is somewhat similar to the one analyzed in Refs. 11 and 12, but we now consider here how the dc magnetic field affects the radiation. Remarkably, and in contrast to common beliefs, a weak magnetic field can enhance the radiation and only a relatively high H_{dc} would suppress it.

The Josephson dynamics in layered superconductors can be described by a set of coupled sine-Gordon equations relating the gauge-invariant phase difference φ_n between the nth and $(n+1)$th layers and electromagnetic fields.[22] These equations can be derived from Maxwell's equations expressing fields and currents in terms of φ_n. The gauge-invariant phase difference is defined as (see, e.g., Ref. 23)

$$\varphi_n = \chi_{n+1} - \chi_n + 2\pi s A_{nz}/\Phi_0, \tag{1}$$

where χ_n is the phase of the superconducting order parameter in the nth layer and A_z is the z component of the vector-potential of the electromagnetic field between the nth and $(n+1)$th layers. The components of the current J_z and J_x flowing in the SJJ have both superconducting and quasiparticle contributions and can be expressed as[6]

$$J_z = J_c \sin\varphi_n + \frac{\sigma_c \Phi_0}{2\pi c s}\frac{\partial \varphi_n}{\partial t},$$

$$J_x = \frac{c\Phi_0}{8\pi^2 \lambda_{ab}^2} p_n + \frac{\sigma_{ab}\Phi_0}{2\pi c}\frac{\partial p_n}{\partial t}, \tag{2}$$

where σ_c and σ_{ab} are quasiparticle conductivities across and along the superconducting planes, λ_{ab} is the in-plane London penetration depth,

$$p_n = \partial\chi_n/\partial x - 2\pi A_x/\Phi_0,$$

and A_x is the x component of the vector-potential. Substituting these formulas in Maxwell's equations and using the relation between φ_n and the y component H_{ny} of the magnetic field between the nth and $(n+1)$th layers,[9,23]

$$H_{ny} = \frac{\Phi_0}{2\pi s}\left(\frac{\partial \varphi_n}{\partial x} + p_n - p_{n+1}\right), \tag{3}$$

we can derive the set of coupled sine-Gordon equations in the form[6,11]

$$\frac{1}{c^2}\frac{\partial^2 \varphi_n}{\partial t^2} = (\alpha\nabla_n^2 - 1)\left(\frac{4\pi\sigma_c}{\varepsilon c^2}\frac{\partial \varphi_n}{\partial t} + \frac{\omega_p^2}{c^2}\sin\varphi_n - \frac{2\pi s}{\Phi_0 \varepsilon}\frac{\partial H_{ny}}{\partial x}\right),$$

$$\left[1 - \left(\frac{\lambda_c}{\gamma s}\right)^2 \nabla_n^2\right] H_{ny} = \left(1 + \frac{4\pi\sigma_{ab}}{\varepsilon\gamma^2\omega_p^2}\frac{\partial}{\partial t}\right)\left(\frac{\Phi_0}{2\pi s}\frac{\partial \varphi_n}{\partial x} - H_{ny}\right), \tag{4}$$

where $\nabla_n^2 f_n = f_{n+1} + f_{n-1} - 2f_n$, and we introduce the capacitive coupling parameter between adjacent superconducting layers α, which in the case of BSCCO can be estimated as $\alpha \sim 0.1$ (see Ref. 11).

We calculate the terahertz radiation produced by uniform Josephson oscillations in a stack of identical junctions. In this case, the gauge-invariant phase difference and magnetic field are the same for any n, $\varphi_n = \varphi$ and $H_{ny} = H_y$. In this case, the second of Eq. (4) reduces to

$$H_y = \frac{\Phi_0}{2\pi s}\frac{\partial \varphi}{\partial x}. \tag{5}$$

We substitute this relation to the first of Eq. (4) and obtain a single sine-Gordon equation for the phase difference in the form,

$$\ddot{\varphi} + \nu_c \dot{\varphi} + \sin\varphi = \varphi'', \tag{6}$$

where we introduce dimensionless time $\tau = t\omega_p$ and coordinate $\eta = x/\lambda_c$, $\nu_c = 4\pi\sigma_c/\varepsilon\omega_p$, $\dot{\varphi} = \partial\varphi/\partial\tau$, and $\varphi' = \partial\varphi/\partial\eta$. The frequency is normalized as $\omega=\omega_J/\omega_p$ and $l=L/\lambda_c<1$. Here we are interested in the frequency range near a cavity resonance when $\omega_J=\pi\omega_p\lambda_c/L$. In dimensionless units it reads $\omega l=\pi$, thus, $\omega\sim\pi/l\gg 1$. We seek a solution of Eq. (6) in the form,

$$\varphi=\varphi_0+\psi, \tag{7}$$

where φ_0 and ψ describe the Josephson oscillations and radiation, respectively, and $|\psi|\ll 1$. Using the relation Eq. (5) between the y component of the magnetic field and the phase difference, we derive boundary conditions for φ_0 in the form $\varphi_0'=h\pm h_I$, at $\eta=\pm l/2$, where

$$H_0=\frac{\Phi_0}{2\pi s\lambda_c},\quad h\equiv\frac{H_{\mathrm{dc}}}{H_0},\quad\text{and}\quad h_I=\frac{l}{2}\left(\frac{J}{J_c}\right)\ll 1. \tag{8}$$

So, the part of the phase difference, which relates to the Josephson oscillations and satisfies the formulated above boundary conditions can be written as

$$\varphi_0=\omega\tau+h\eta+h_I\eta^2l^{-1}, \tag{9}$$

where the last term in the right-hand side is small. Substituting now the sum $\varphi=\varphi_0+\psi$ in Eq. (6), we obtain the equation for ψ in the limits $|\psi|\ll 1$ and $\omega\gg 1$,

$$\psi''-\ddot{\psi}-\nu_c\dot{\psi}=\sin(\omega\tau+h\eta)+\nu_c\omega-2h_Il^{-1}, \tag{10}$$

where we neglect the small terms $(h_I\eta^2l^{-1}+\psi)$ in the argument of the sine. In the resistive state $\Delta V=Js/\sigma_c$ and we can verify that the last two terms cancel in the right-hand side of Eq. (10). We substitute the solution ψ in the form

$$\psi(\tau,\eta)=\mathrm{Re}[\psi_\omega(\eta)\exp(-i\omega\tau)] \tag{11}$$

in Eq. (10) and obtain for the amplitude ψ_ω the equation[11]

$$\psi_\omega''+\kappa^2\psi_\omega=ie^{-ih\eta}, \tag{12}$$

where

$$\kappa=\sqrt{\omega^2+i\nu_c\omega}. \tag{13}$$

According to Ref. 12, the boundary conditions for ψ have the form

$$\psi_\omega'=\pm i\zeta\psi_\omega,\quad x=\pm l/2, \tag{14}$$

where

$$\zeta=\frac{L_z\omega^2}{2\varepsilon\lambda_c}\left[1-\frac{2i}{\pi}\ln\left(\frac{5.03\sqrt{\varepsilon}\lambda_c}{\omega L_z}\right)\right]. \tag{15}$$

In the case of BSCCO, $\lambda_c\sim 100$ μm, $\varepsilon\sim 10$–20, and $|\zeta|\lesssim 10^{-3}\omega^2$ if $N\lesssim 10^3$.

A solution of Eq. (12) with its boundary conditions is

$$\psi_\omega=\psi_\omega^{\mathrm{rad}}+\frac{i\exp(-ih\eta)}{\kappa^2-h^2}. \tag{16}$$

where

$$\psi_\omega^{\mathrm{rad}}=C_1e^{i\kappa\eta}+C_2e^{-i\kappa\eta}, \tag{17}$$

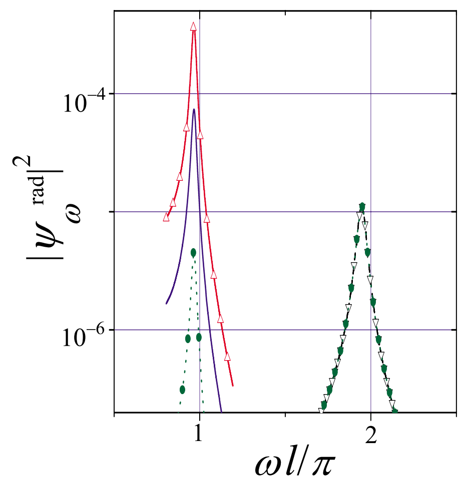

FIG. 2. (Color online) Dimensionless radiation power $|\psi_\omega^{\mathrm{rad}}(l/2)|^2$ versus ω for different values of the dc magnetic field: $H_{\mathrm{dc}}=0$ (black dashed line with down triangles), $0.1H_0$ (dotted line with green circles), $0.4H_0$ (blue solid line), $0.9H_0$ (red line with up triangles). The sample parameters used here are: $L=0.25\lambda_c$, $\lambda_c=100$ μm, $D=1.56$ nm, $N=3\times 10^3$, $\varepsilon=12$, and $\nu_c=0.002$.

$$C_{1,2}=\frac{\mp(h\kappa\mp\zeta^2)\sin\dfrac{(\kappa\mp h)l}{2}+i\zeta(\kappa\mp h)\cos\dfrac{(\kappa\mp h)l}{2}}{2(\kappa^2-h^2)\left[\kappa\cos\dfrac{\kappa l}{2}-i\zeta\sin\dfrac{\kappa l}{2}\right]\left[i\kappa\sin\dfrac{\kappa l}{2}-\zeta\cos\dfrac{\kappa l}{2}\right]}, \tag{18}$$

The first term, $\psi_\omega^{\mathrm{rad}}$, in Eq. (16) describes the radiation, while the second term is a correction to the Josephson oscillations accounting for H_{dc}.

The radiation power P from SJJ is a sum of the powers from the left P_- and the right P_+ sides of the sample, which can be expressed in terms of the Pointing vector $(c/4\pi)\mathbf{E}\times\mathbf{H}$ as

$$P_\pm=\frac{cL_ysN}{4\pi}E_z^{\mathrm{rad}}(\pm l/2)H_y^{\mathrm{rad}}(\pm l/2), \tag{19}$$

where E_z is the electric field across the SJJ, L_ysN is half of the SJJ side surface, and the superscript "rad" means that we take into account only oscillating parts of the electromagnetic fields. In dimensionless notation, Eq. (5) reads $H_y=H_0\varphi'$. Using this formula and Maxwell's equation $c\nabla\times\mathbf{E}=-\partial\mathbf{H}/\partial t$, we find the relation between the z component of the electric field and the phase difference in the form[9] $E_z=\lambda_c\omega_pH_0\dot{\varphi}/c$. Substituting these relations in Eq. (19) and using the boundary conditions Eq. (14), we derive an approximate formula for the radiation power,[12]

$$P_\pm\approx\frac{L_y\Phi_0^2N^2\omega_J^3}{64\pi^3c^2}\left|\psi_\omega^{\mathrm{rad}}\left(\pm\frac{l}{2}\right)\right|^2, \tag{20}$$

where the N^2-dependence indicates a *super-radiance* regime.

The radiation powers to the right and to the left of the sample are equal. The dependence of the dimensionless radiation power $|\psi_\omega(l/2)|^2$ on ω is shown in Fig. 2 for different values of H_{dc}. The power $P_\pm(\omega)$ has a set of resonance peaks at $\kappa l/2 \approx \pi n$, with an intensity rapidly decreasing with n. Applying a dc field results in the growth of peaks with odd n, which are absent when $h=0$, and does not affect the peaks with even n.

At the first resonance

$$\omega_{\mathrm{res}} \approx \frac{\pi}{l} - \frac{2}{\pi}\mathrm{Im}(\zeta),$$

we derive, using Eq. (16) and the expressions for $C_{1,2}$,

$$\psi_\omega^{\mathrm{rad}}\left(\frac{\pm l}{2}\right) = \frac{-4ihl^2\cos\dfrac{hl}{2}}{(\pi^2 - h^2l^2)[\pi\nu_c + 4\,\mathrm{Re}(\zeta)]}. \tag{21}$$

Note that *the peak height increases linearly with the magnetic field* if $(hl/\pi)^2 \ll 1$. It *grows sharply* when hl is close to π. However, the results obtained here are valid only if $|\psi| \ll 1$ and we restrict our consideration to the limit $h^2 \ll \pi^2/l^2$. The peak width is determined by the dissipation ν_c and radiation losses $\mathrm{Re}(\zeta) \propto N$. If the radiation losses become much larger than ν_c [here $N \gg N_c = \nu_c \varepsilon L^2/(2\pi s \lambda_c)$], the radiation power becomes independent of N, similar to the case of uniform or artificially modulated SJJ (Refs. 11 and 12); we estimate $N_c \approx 100$ for the values used to calculate the curves shown in Fig. 2. Comparing Eqs. (20) and (21) with the maximum radiation power for uniform and artificially modulated SJJ,[11,12] we conclude that the application of H_{dc}: (i) enhances the radiation power if

$$H_{\mathrm{dc}} > H_e = \frac{3\pi^2 L_z H_0}{8\varepsilon L}, \tag{22}$$

and (ii) is more effective than the linear modulation of J_c by a factor of g (the ratio of J_c on the left and the right edges of the SJJ) if

$$H_{\mathrm{dc}} > H_g = \frac{2\lambda_c H_0 (g-1)}{L(g+1)}. \tag{23}$$

We estimate $H_0 \approx 20$ Oe, $H_e \approx 1.2$ Oe, and $H_g \approx H_0$, when $g=1.3$, for the same parameter values as in Fig. 2.

According to our calculations, the application of the dc magnetic field increases the terahertz radiation from the SJJ by orders of magnitude as compared to the uniform sample in zero applied field. Note, however, that experimentally, additional contributions to the bias current may exist, causing a spatial variation in the bias current density along the y direction. Also, incidental inhomogeneities may also exist in real SJJs. These effects may increase the radiation at $H=0$ and mask the effect of the magnetic field.

III. STABILITY OF THE SYNCHRONIZED OSCILLATIONS

We analyze the stability of the synchronized regime with respect to z-dependent perturbations, which could destroy the synchronization in different junctions. First, we rewrite Eq. (4) in dimensionless units,[11]

$$\ddot{\varphi}_n = (\alpha\nabla_n^2 - 1)(\nu_c\dot{\varphi}_n + \sin\varphi_n - h'_{ny}),$$

$$(\nabla_n^2 - \hat{T}_{ab})h_{ny} + \hat{T}_{ab}\varphi'_n = 0, \tag{24}$$

where $\nu_{ab} = 4\pi\sigma_{ab}/(\gamma^2\varepsilon\omega_p)$, $\gamma = \lambda_c/\lambda_{ab}$ is the anisotropy ratio, and

$$\hat{T}_{ab} = \left(\frac{\gamma s}{\lambda_c}\right)^2\left(1 + \nu_{ab}\frac{\partial}{\partial t}\right). \tag{25}$$

In the case of BSCCO, $\nu_{ab} \sim 0.1$.

We write φ_n as

$$\varphi_n = \omega\tau + h\eta + \psi + \theta_n, \tag{26}$$

where θ_n is an infinitesimally small perturbation depending on the position of the junction z_n. In the limit $\omega \gg 1$, we neglect higher ($m\omega$ with $m>1$) harmonics and present θ_n in the form[11]

$$\theta_n = \sum_q [\theta_q + \theta_q^+ e^{i\omega\tau} + \theta_q^- e^{-i\omega\tau}]\sin(qn)e^{-i\Omega\tau}, \tag{27}$$

where $q = \pi k/(N+1)$, $k = 1, 2, \ldots, N$; $|\Omega(q)| \ll 1$; and the uniform (along the z axis) oscillations are stable if $\mathrm{Im}[\Omega(q)] < 0$ for all q. We analyze the case $q \gg \pi/N$, since the inverse limit corresponds to a uniform solution. The perturbation of the magnetic field has a similar form as in Eq. (27). Substituting Eqs. (26) and (27) in Eq. (24), excluding the perturbation of the field, separating the terms with the same frequencies, and taking into account that $|\psi|$, $|\theta_n| \ll 1$, we derive

$$G^2(\Omega)\theta''_q + [\kappa_a^2(\Omega) + \tilde{\psi}_\omega]\theta_q = \frac{\theta_q^+ e^{-ih\eta} + \theta_q^- e^{ih\eta}}{2}, \tag{28}$$

$$G^2(\Omega \mp \omega)(\theta_q^\pm)'' + \kappa_a^2(\Omega \mp \omega)\theta_q^\pm = \frac{\theta_q e^{\pm ih\eta}}{2}, \tag{29}$$

where

$$G^2(\omega) = \frac{1 - i\nu_{ab}\omega}{1 - i\nu_{ab}\omega + \ell^2\tilde{q}^2}, \quad \kappa_a^2(\omega) = \frac{\omega^2}{1 + \alpha\tilde{q}^2} + i\nu_c\omega, \tag{30}$$

$$\tilde{\psi}_\omega = \frac{\mathrm{Re}(\psi_\omega)}{2}\sin(h\eta) + \frac{\mathrm{Im}(\psi_\omega)}{2}\cos(h\eta), \quad \ell = \frac{\lambda_c}{\gamma s}, \tag{31}$$

where $\tilde{q}^2 = 2(1 - \cos q)$. In Eq. (29), we neglect terms of the order of $|\psi_\omega|$ compared to $\omega \gg 1$. Under these assumptions, the boundary conditions to these equations are the same as in Ref. 11,

$$\frac{\theta'_q}{\theta_q} = \pm\frac{\chi(\Omega)}{G^2(\Omega)}, \quad \frac{(\theta_q^\beta)'}{\theta_q^\beta} = \pm\frac{\chi(\Omega - \beta\omega)}{G^2(\Omega - \beta\omega)}, \tag{32}$$

$$\chi(\omega) = \frac{\omega^2}{1 + \alpha\tilde{q}^2}\frac{s}{\varepsilon q\lambda_c}, \quad \eta = \pm\frac{l}{2}, \tag{33}$$

where the index β is either + or −.

Now we analyze Eqs. (28) and (29). First, notice that the characteristic spatial scale of variation of θ_q is

$$\frac{1}{|\Omega G(\Omega)|} \approx \frac{\sqrt{1+\ell^2\tilde{q}^2}}{|\Omega|} \gg l,$$

as it follows from Eq. (28). Thus, this scale is much larger than the sample size. Thus, $\theta_q(\eta)$ is almost constant and we can find solutions of Eq. (29) explicitly. For simplicity, we neglect $\Omega \ll 1$ compared to $\omega \gg 1$ in Eqs. (29) and (32). We rewrite Eq. (29) in the form

$$(\theta_q^-)'' + k^2(\omega)\theta_q^- = \frac{\theta_q e^{-ih\eta}}{2G^2(\omega)}, \tag{34}$$

where

$$k(\omega) = \kappa_\alpha(\omega)/G(\omega).$$

We can estimate $|k| \approx \sqrt{1+\ell^2\tilde{q}^2}\omega \gg 1$. We consider here the field range of the order of $hl \sim \pi/2 \sim 1$. Then, in the limit $k \gg h$, we derive from the last equation $\theta_q^- \exp(ih\eta) \approx \theta_q V(\eta)$, where

$$V = \frac{1}{2\kappa_\alpha^2(\omega)}\left\{1 - \frac{\chi(\omega)\cos k\eta}{G^2(\omega)\left[G(\omega)\kappa_\alpha(\omega)\sin\frac{kl}{2} + \chi(\omega)\cos\frac{kl}{2}\right]}\right\}.$$

We derive a similar expression for θ_q^+. Substituting these results in Eq. (28), we have

$$\theta_q'' + \frac{\kappa_\alpha^2(\Omega) + \tilde{\psi}_\omega(\eta) - \mathrm{Re}[V(\eta)]}{G^2(\Omega)}\theta_q = 0. \tag{35}$$

This equation can be solved considering the coordinate dependence of θ_q as a small perturbation. Thus, integrating Eq. (35) with the boundary conditions [Eq. (32)], we derive a dispersion relation for $\Omega(q)$ in the form

$$\frac{\Omega^2}{1+\alpha\tilde{q}^2} + i\nu_c\Omega + W_1(\omega) + W_2(\omega) \approx \frac{\alpha\tilde{q}^2\omega^2 - ah^2}{2\omega^4}, \tag{36}$$

where $a \lesssim 1$,

$$W_1 = \frac{1}{l}\mathrm{Im}\left[C_1\frac{\sin\frac{(\kappa+h)l}{2}}{\kappa+h} + C_2\frac{\sin\frac{(\kappa-h)l}{2}}{\kappa-h}\right], \tag{37}$$

$$W_2 = \mathrm{Re}\left\{\frac{\chi(\omega)}{l\kappa_\alpha^3(\omega)G(\omega)\left[\kappa_\alpha(\omega)G(\omega) + \chi(\omega)\cot\left(\frac{kl}{2}\right)\right]}\right\}, \tag{38}$$

and we assume that $N \ll \varepsilon\lambda_c/s \approx 5\times10^5$. The obtained Eq. (36) for Ω allows us to write down the stability criterion for the synchronized Josephson oscillations explicitly. The condition $\mathrm{Im}[\Omega(q)] < 0$ is true if the free term of the quadratic equation Eq. (36) is negative. Thus, the oscillations in different junctions are synchronized if

$$W_1(\omega,h) + W_2(\omega,\tilde{q}) < \frac{\alpha\tilde{q}^2\omega^2 - ah^2}{2\omega^4} \tag{39}$$

at any value of $\tilde{q}$.

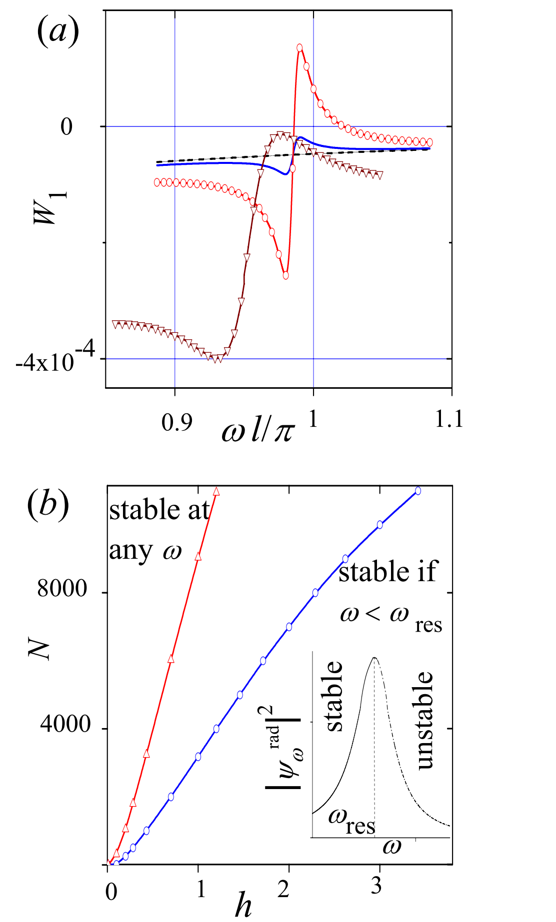

FIG. 3. (Color online) (a) Function $W_1(\omega)$ near the resonance: $H_{dc}=0$, $N=10^3$ (black dashed line), $H_{dc}=0.2H_0$, $N=10^3$ (blue solid line), $H_{dc}=0.5H_0$, $N=10^3$ (red line with circles), $H_{dc}=H_0$, $N=4.5\times10^3$ (green line with triangles). Other parameters are the same as in Fig. 2. (b) Stability margins in the (N,h)-plane for $l=0.25$ (blue line with circles) and $l=0.4$ (red line with triangles). In the inset: schematics of the stable and unstable regions of the resonance peak.

If the term $W_1 < 0$, it stabilizes the uniform oscillations due to radiation coupling. The term W_2 leads to instability due to the excitation of the Fiske resonance.[11] As in the case of zero magnetic field, we find that *both* dissipation and charge-neutrality-breaking stabilize the super-radiance regime. The magnetic field reduces slightly the effect of the charge-neutrality-breaking. However, according to our numerical estimates, the coefficient a, in Eq. (36), is small.

The magnetic field does not practically affect W_2, but it significantly reduces the radiation coupling between junctions. Indeed, the value W_1 changes its sign, becomes zero in the main approximation at the resonance point [see Fig. 3(a)], and stabilization due to radiation coupling can only occur due to the next-order terms with respect to ζ. Thus, at sufficiently high magnetic fields and low number of junctions, the resonance radiation is stable at frequencies lower than the resonance frequency and it becomes unstable if $\omega > \omega_{res}$; see inset in Fig. 3(b). For lower fields and for a SJJ with a large number of junctions, the resonance peak is stable for both higher and lower frequencies.

The margins and regions of stability are shown in Fig. 3(b) in the plane (h,N) for two different values of l. The radiation is stable for all frequencies above the lines $N(h)$ and only for $\omega < \omega_{\mathrm{res}}$ below these curves. The results presented here show that, for a given sample, the in-phase oscillations are stable only at low magnetic fields. With increasing h, the stability of the synchronized regime decreases and the stable regimes occur only for the low frequency modes, $\omega < \omega_{\mathrm{res}}$.

As it follows from these results, we can switch off and on radiation by varying either the magnetic field or the bias current (i.e., ω_J). The region of full stability decreases when increasing the ratio L/L_z. We can assume that the instability when $\omega > \omega_{\mathrm{res}}$ arises due to the negative differential resistivity (NDR) of the IV curve near the peak of the radiation power. Indeed, parts of the IV curve with the NDR were observed near radiation peaks in BSCCO mesas.[8] The IV-curve calculation in Ref. 11 also predicts such IV-curve singularities. Our considerations are invalid if $hl \gtrsim \pi$. Perhaps the synchronization of the oscillations in different BSCCO junctions will be destroyed at higher H_{dc} since a vortex lattice enters the sample. However, this occurs at a field $H_{\mathrm{dc}} \gg H_{c1}$ due to the small thickness ($L \ll \lambda_c$) of the SJJ and the current flowing in the sample along the c direction.

IV. CONCLUSION

The application of a moderate dc magnetic field can significantly increase synchronized terahertz emission from SJJ. The magnetic field affects identically all the junctions in the SJJ, which is a considerable advantage of our proposal as opposed to the use of SJJ with artificially modulated properties. However, the dc magnetic field reduces the stability of the uniform Josephson oscillations.

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